LS-104
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Correction of Closed Orbit Distortions in the Horizontal Direction

I.

Many computer programs with a variety of algorithms exist for controlling the closed orbit in synchrotrons. One of the more recent reports on this subject explains how the closed orbit was established in the Fermilab Tevatron on "Day One" and how it is manipulated during routine operations.[1] In most synchrotrons, the beam position monitors and the steering dipoles are located side by side and algorithms such as the familiar three-bump orbits are easy to understand. When a beam is kicked at a location "A", the resulting beam displacement at another location "B" downstream is proportional to $\sin(\Delta\psi)$ where $\Delta\psi$ is the betatron phase advance from "A" to "B". It is therefore reasonable to have the phase advance between two adjacent position monitors somewhere around 45° to 90° and to make it more or less uniform around the ring. The commonly used FODO-type lattice with the phase advance of 60° to 90° per cell satisfies these conditions and many computer programs for controlling the closed orbit are tailored to such an arrangement.

When judged from this consideration, the Chasman-Green lattice chosen for the 7-GeV APS synchrotron and the arrangement of position monitors and steering dipoles in it are quite abnormal. Because of the importance of closed orbit deviation within (very strong) sextupoles, position monitors of both polarities are placed at the sextupole locations. In the vertical direction, this may be regarded as "natural" since steering dipoles are also located at the same place, i.e., inside each sextupole. Furthermore, the phase distance between adjacent position monitors are surprisingly uniform in the vertical direction as one can see in Fig. 1. No serious problems are anticipated in controlling the vertical closed orbit with the standard algorithms which are being used for the FODO-type arrangements. Horizontal direction is an entirely different story.

One sees in Fig. 2 that, in the horizontal direction, the phase distance from one position monitor to the next is quite irregular; it is only 7° or as much as 100°. Similar irregularities exist in phase distance of steering elements, particularly if the trim windings in dipoles (B_U and B_D in Fig. 2) are not used to control the horizontal closed orbit. In order to establish a horizontal closed orbit on "Day One" and, subsequently, to reduce its deviation as much as possible, a series of new procedures tailored to this abnormal situation may become necessary. This is now being investigated by W. Chou. Extensive numerical studies on orbit corrections with various combinations of steering dipoles have been performed by Y. Jin using computer programs such as PETROC, PETROS, RACETRACK and MAD.

The scope of this note is rather modest in comparison. Based on a simple model, a study has been made to find out statistically how much kick angle is needed by each steering element and how much residual closed orbit deviation should be expected when the closed orbit is steered to go through the center of seven position monitors (M_2 through M_8) in each cell. Seven independent kicks are supplied by two trim dipoles B_U and B_D, and six steering elements (H_1 through H_6) with H_3 and H_4 assumed to have the same kick angle. If it is necessary to remove H_3 to make a space there for a correction skew quadrupole (in every other cell), the kick angle of H_4 would have to be doubled.

II.

A very simple statistical argument can be used to make an approximate estimate of the rms kick angle required at each steering element.

The expected overall effect of quadrupole misalignments can be represented by the quantity

$$\sum_i [\beta_x (B'\ell/B\rho)^2 \Delta^2]_i = 11.6 \text{ m}^{-1} \Delta_x^2/\text{half cell} \tag{II.1}$$

where the sum is for five quadrupoles in half cell and Δ_x is the rms of quadrupole horizontal misalignments Δ_i. If three steering dipoles (H_1, H_2, H_3) are used in each half cell to correct for this effect, one must have

$$\sum \beta_x \theta^2 = 35.9 \text{ m} < \theta^2 > = 11.6 \text{ m}^{-1} \Delta_x^2 \tag{II.2}$$

or

$$\sqrt{<\theta^2>} = 0.57 \text{ m}^{-1} \Delta_x .$$

If Δ_x is 0.5 mm (which is much more than the design goal of 0.1 mm), the rms value of required kick angle at each steering element is 0.29 mrad. Since each steering element is capable of kicking the 7-GeV beam up to ±1.3 mrad, this is quite comfortable. With only two steering dipoles (H_2, H_3) in each half cell, the required kick will be

$$\sqrt{\langle\theta^2\rangle} = 0.89\ m^{-1}\Delta_x$$

which may be too large to feel completely safe if Δ_x is indeed as large as 0.5 mm. For the design goal of Δ_x = 0.1 mm, there will be no problem even with two steering elements/half cell. It should be obvious that, in discussing closed orbit deviations caused by quadrupole misalignments, the beam motion is assumed to be linear. Specifically, this implies that all sextupoles are off when one is trying to control the closed orbit. If sextupoles are on (for whatever the reason), the adjustment of closed orbit must be done in several small steps.

Since there are nine position monitors in each cell, there will be some residual closed orbit deviations at all position monitors when only four or six independent kicks are used in one cell. The argument presented here cannot predict how much residual deviations one must expect. In the following section, seven independent kicks are used in each cell in order to have a perfect closed orbit at seven position monitors, M_2 through M_8, and the residual orbit deviations are calculated at other locations where the beam position cannot be observed.

III.

It is easy to see that the dominant source of horizontal orbit deviations is the quadrupole misalignment. For the 7-GeV ring, the expected horizontal deviation at an arbitrary point is given by

$$< \xi_x^2 + \eta_x^2 > = 590 \ m^{-1} \ \Delta_x^2 \tag{III.1}$$

Normalized coordinates are used here in order to make the transfer from one point to another a simple rotation with the betatron phase advance as the rotation angle:

$$\xi_x = x/\sqrt{\beta_x} \quad \text{and} \quad \eta_x = \sqrt{\beta_x}\, x' + (\alpha_x/\sqrt{\beta_x})x \tag{III.2}$$

There are 7x40 = 280 unknowns (kick angles) in the ring to satisfy the equal number of conditions (i.e., deviations at 280 monitors). Since the motion is assumed to be linear, solving the 280 equations is accomplished by inverting a (280 × 280) matrix which is not an easy task for a simple program. Numerical inversion of such a matrix often leads to many difficulties when single precisions are used.

In order to avoid this difficulty, a simple model has been used. A closed orbit (x_0, x_0') was selected randomly at the center "C" of an insertion (see Fig. 2) in accordance with (III. 1). Misalignments of thirteen quadrupoles downstream of "C" were chosen randomly using the same value of Δ_x

and the resulting orbit deviations were calculated at nine monitors; M_2 through M_8 downstream of "C" plus M_2 and M_3 of the next cell. Nine independent kicks were then found such that the corrected closed orbit deviation is zero at these nine position monitors. Nine kicks are at H_6 (which is upstream of "C"), H_1 through H_6 (with $H_3 = H_4$), H_1 of the next cell and two trim windings B_U and B_D. The kick angles for the first two elements, H_6 and H_1, are understandably very large since, acting alone, they must eliminate the deviations at M_2 and M_3. These large angles are not real; if effects of upstream elements were taken into account, the required angle would become much smaller. The proper values are found at the last two steering elements designated as H_6' and H_1' in Fig. 2. When all kick angles are found, the closed orbit deviations at other places (where there are no monitors) can be calculated.

For one hundred samples randomly chosen, the results are tabulated below. All quantities are linearly proportional to Δ_x so that the units for angle and orbit deviation are mrad and mm, respectively, when Δ_x is measured in mm.

A. Kick angles

steering dipoles	max./Δ_x	rms/Δ_x
H_2	0.62	0.22
BU	1.20	0.42
H_3 & H_4	1.56	0.52
BD	1.14	0.41
H_5	0.58	0.21
H_6'	1.68	0.57
H_1'	1.57	0.55

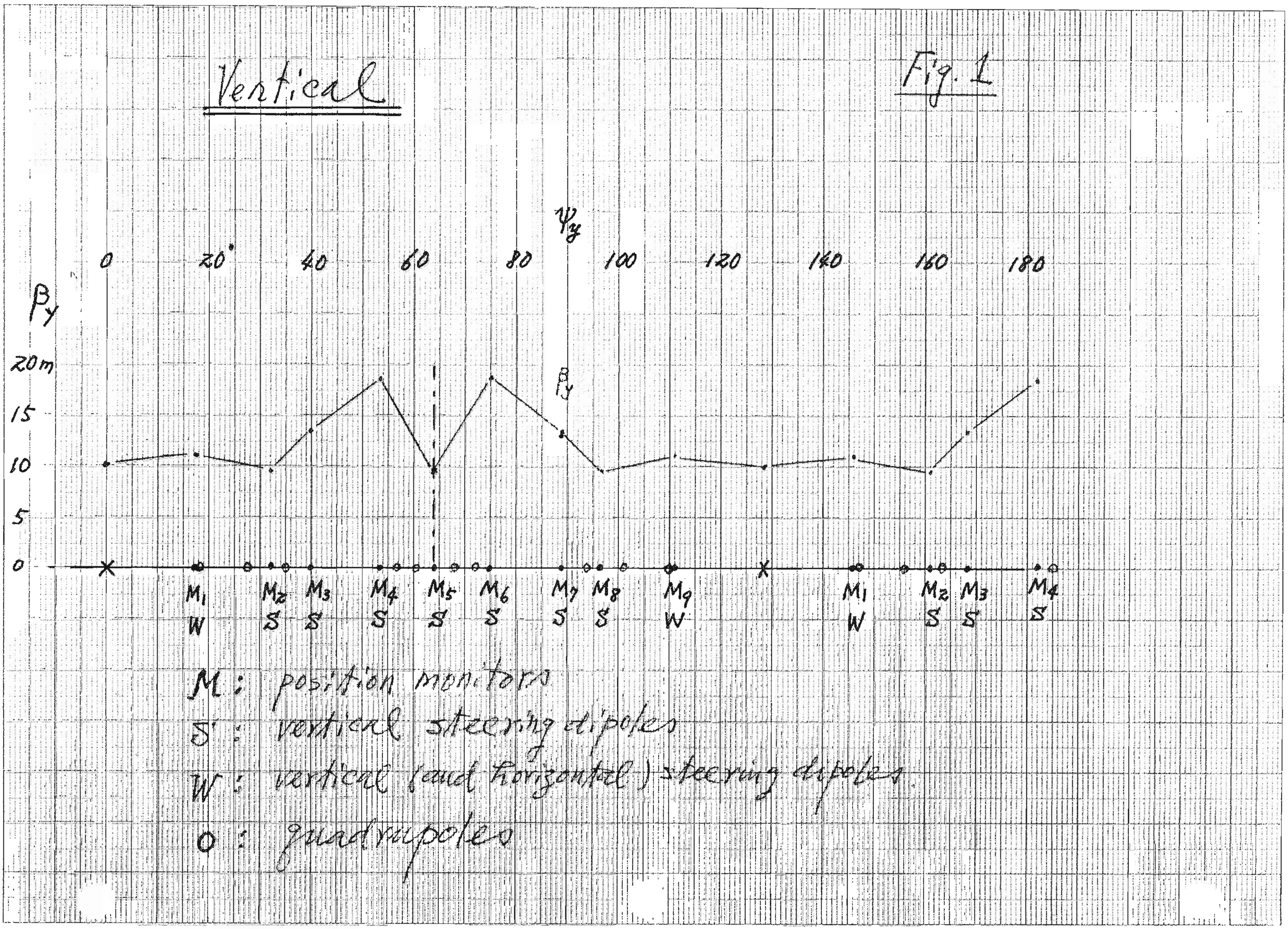
B. Residual closed orbit deviation

location	max./Δ_x	rms/Δ_x
BU	1.12	0.39
H_3	0.76	0.25
H_4	0.84	0.25
BD	1.05	0.38
H_6'	0.68	0.25
H_1'	0.64	0.24

At 7 GeV, the maximum capability of steering dipoles (H_1 to H_6) is ±1.3 mrad while it is ±5.2 mrad for BU and BD. Based on this study alone, one may conclude that Δ_x = 0.25 mm or less can be accommodated with the present steering system. The question remains, however, as to whether one should be able to establish a measurable closed orbit on "Day One" without too much struggling and what procedures are best suited for this purpose.

Reference

1. R. Raja, A. Russell and C. Ankenbrandt, Nucl. Inst. Meth. **A242**, 15 (1985).


Vertical
Fig. 1
ψ_y
0
20°
40
60
80
100
120
140
160
180
β_y
20m
15
10
5
0
β_y
M1 W
M2 S
M3 S
M4 S
M5 S
M6 S
M7 S
M8 S
M9 W
M1 W
M2 S
M3 S
M4 S
M : position monitors
S : vertical steering dipoles
W : vertical (and horizontal) steering dipoles
O : quadrupoles

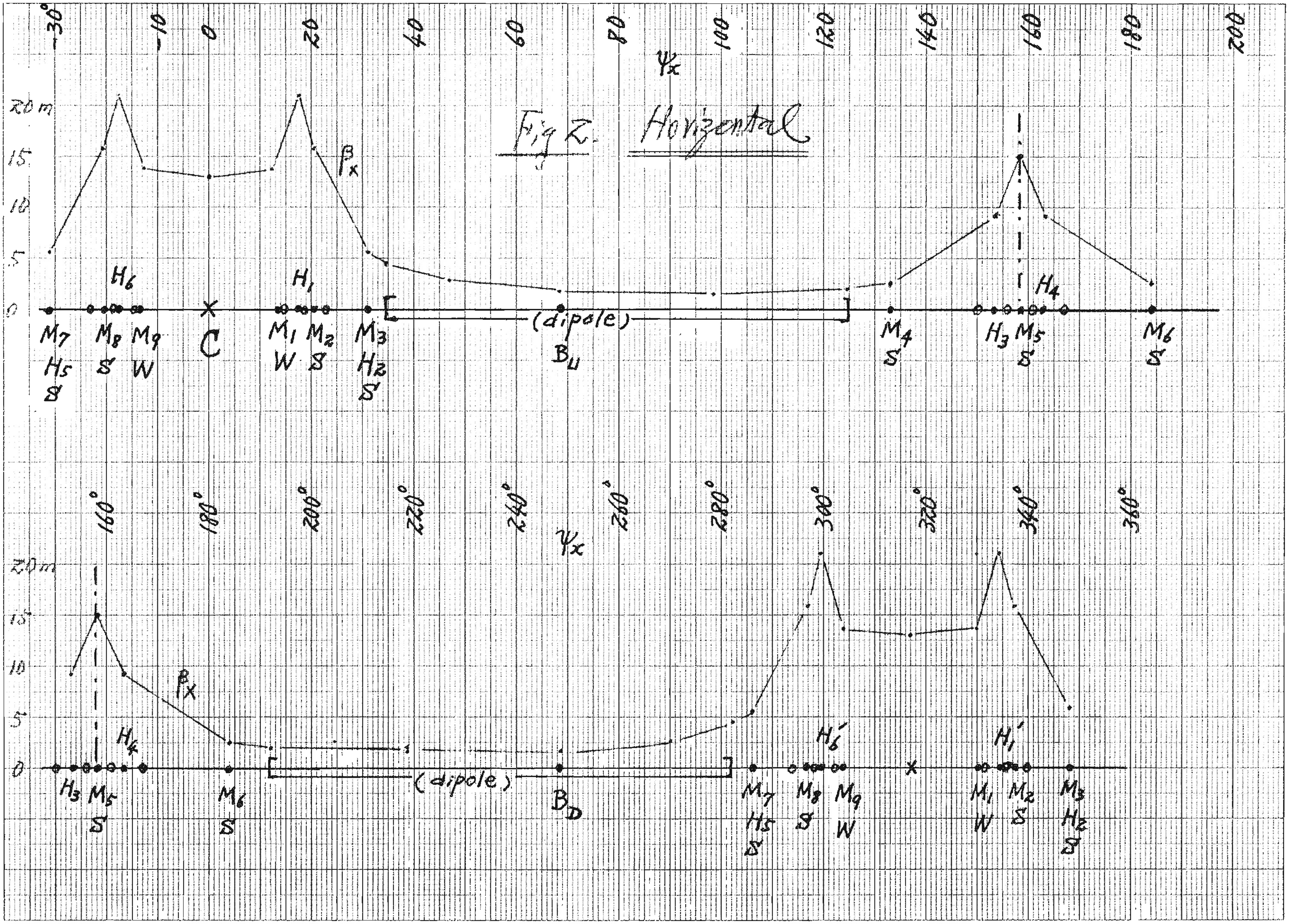


Fig 2. Horizontal
ψ_x
β_x
-30 -10 0 20 40 60 80 100 120 140 160 180 200
20 m
15
10
5
0
H_6
H_1
H_4
(dipole)
M_7 H_5 S
M_8 S
M_9 W
C
M_1 W
M_2 S
M_3 H_2 S
B_U
M_4 S
H_3
M_5 S
M_6 S
160° 180° 200° 220° 240° 260° 280° 300° 320° 340° 360°
H_1'
H_6'
B_D